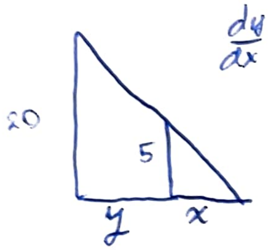
Solutions

March 14, 2025

Math 1A Worksheet #20

Name: ______________________________

1. A street light is mounted 20 feet above a ground at the top of a pole. A 5 foot-tall teenager runs towards the pole at a rate of 7 feet per second. At what rate is the length of her shadow shrinking while she is 28 feet from the pole? (your answer should be negative)



$\frac{dy}{dx} = -7$ ft/s

$\frac{x}{5} = \frac{x+y}{20} \to 15x = 5y$

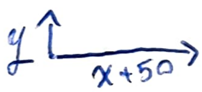
$\to 3x = y$

$\cancel{18\frac{dx}{dt}}$

$3\frac{dx}{dt} = \frac{dy}{dt}$

$\frac{dx}{dt} = -7/3$ ft/s.

2. A red car is positioned 50 kilometers due east of a blue car. Suddenly, both cars start moving: the red car moves at a constant rate of 20 km/h east, while the blue car moves at a constant rate of 30 km/h north. At what rate is the distance between the two cars increasing 3 hours after the two cars start moving?



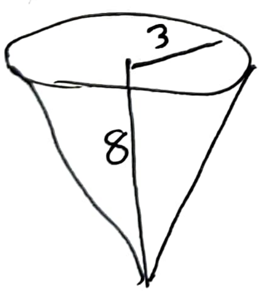
$D \cancel{d} = \sqrt{(x+50)^2 + y^2}$

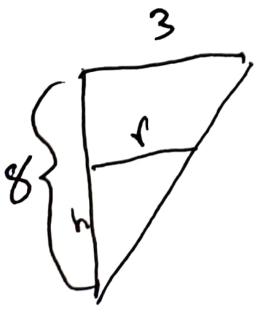
$\frac{dD}{dt} = \frac{1}{2\sqrt{\cdots}}\left(2(x+50)\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$

at 3 hours, $x = 60$, $y = 90$

$\frac{dD}{dt} = \frac{1}{2\sqrt{110^2+90^2}}\left(2(110)(20) + 2(90)(30)\right)$ km/hr

3. A fluid is being pumped into an inverted conical tank at a constant rate of $10,000\ \text{cm}^3/\text{min}$. Suppose that the tank has height 8 m and the diameter of the top is 6 m. What is the rate at which the water level is rising when the height of the water is 4 m?





4. Estimate the following quantities using linear approximation via a tangent line. A hint is provided for the first part.

(a) $e^{-0.1}$ (use the tangent line to $y = e^x$ at $(x, y) = (0, 1)$)

(b) $\sin(3.24)$ (recall that $\pi \approx 3.14$).

(c) $\sqrt{9.001}$

(d) 1.999^3

A.) $y' = e^x$, $y'(0) = 1$, $\leadsto$ tangent $y = x + 1$; $e^{-0.1} \approx 0.9$

B) $y = \sin(x)$; $y'(\pi) = \cos(\pi) = -1$ $\leadsto$ tangent $y = -x + \pi$ $\quad \sin(3.24) \approx 0.1$

C.) $y = \sqrt{x}$; $y'(x) = \frac{1}{2\sqrt{x}}$; $y'(9) = 1/6$; tangent $y = \frac{1}{6}(x-9) + 3$; $\sqrt{9.001} \approx 3 + \frac{1}{6000}$

d.) $y = x^3$; $y'(x) = 3x^2$; $y'(2) = 12$; tangent ~~$y = 2(x-2)+4$~~ $y = 12(x-2) + 8$; $1.999^3 \approx 8 - 0.012$

5. Let u, v be functions of x. Recall that the differential dx is defined as an independent variable, and the differential du is defined by the equation $du = u'(x)dx$. So du depends on x and dx. Prove the following identities.

(a) $d(u + v) = du + dv$ $\quad$ Sum Rule

(b) $d(uv) = v\,du + u\,dv$ $\quad$ Product Rule

(c) $d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$. $\quad$ Quotient Rule.